

Special Semester in Several Complex Variables

Schedule (First Week)

Regular Seminar:

1. Title: Holomorphic Isometric Embedding of Complex Manifolds (1).

Speaker: Dr. Hao Yihong (郝毅红) (Institute of Math., AMSS, CAS).

Time: March 3(Monday), 15:00—17:00.

Place: Room 712 at Si Yuan Building.

Abstract: Whether a Riemann manifold can be isometrically embedded in a space of constant curvature is a classical problem. In Kähler geometry, Calabi gave a complete answer to the problem of the existence and uniqueness of holomorphic isometric embedding from complex manifolds with real analytic Kähler metrics into complex space forms. Afterwards, there appeared many studies such as the metric rigidity between general bounded symmetric domains. In this report, we will introduce Calabi's results and original methods described in his important article (Ann. of Math. 1953).

2. Title: The Levi Problem in Complex Spaces.

Speaker: Li Zhenqian (李震乾) (Institute of Math., AMSS, CAS).

Time: March 4 (Tuesday), 9:30—11:30.

Place: Room 709 at Si Yuan Building.

Abstract: We will discuss the Levi problem in complex spaces and relations with envelope of holomorphy.

3. Title: Multiplier ideal sheaf in analytic and algebraic geometry.

Speaker: Dr. Li Jun (李军) (Peking University).

Time: March 5 (Wednesday), 15:00—17:00.

Place: Room 712 at Si Yuan Building.

Abstract: Multiplier ideal sheaf is a very useful tool in analytic and algebraic geometry. On the analytic side, one can associate a multiplier ideal to a plurisubharmonic function, a singular Hermitian metric etc. On the algebraic side, one can define multiplier ideal for a \mathbb{Q} -effective divisor, an ideal sheaf or a linear series. I'll discuss these concepts, their relationship and some applications.

Working Seminar:

- 4. Title:** Positivity of vector bundles and vanishing theorems.
Speaker: Meng Xiankui (孟宪奎) (Institute of Math., AMSS, CAS).
Time: March 6 (Thursday), 14:00—16:00.
Place: Room 709 at Si Yuan Building.

Abstract: We first give a treatment of complex differential geometry leading up to the Bochner-Kodaira-Nakano identity. Then the positivity concepts for vector bundles will be introduced. Based on these preparations and the basic results from Hodge theory, a few vanishing theorems for Hermitian vector bundles will be proved.

Special Semester in Several Complex Variables

Schedule (Second Week)

Regular Seminar:

- 1. Title:** Oka manifolds.
Speaker: Prof. J.E. Fornæss (Univ. of Michigan).
Time: March 10 (Monday), 15:00—17:00.
Place: Room 703 at Si Yuan Building.

Abstract: I will talk about recent joint work with Erlend Fornæss Wold. Oka manifolds appear naturally in complex analysis. We show that they also appear naturally in complex dynamics.

- 2. Title:** Differential Harnack Estimates on Manifolds.
Speaker: Dr. Niu Yanyan (牛艳艳) (Capital Normal University).
Time: March 11 (Tuesday), 9:30—11:30.
Place: Room 712 at Si Yuan Building.

Abstract: In this talk, we will review the sharp estimates of Li-Yau-Hamilton type for solutions to geometric evolution equations on Riemannian manifolds and Kähler manifolds.

- 3. Title:** Multiplier ideal sheaf in analytic and algebraic geometry (2).
Speaker: Dr. Li Jun (李军) (Peking University).
Time: March 12 (Wednesday), 9:30—11:30.
Place: Room 703 at Si Yuan Building.

Abstract: Multiplier ideal sheaf is a very useful tool in analytic and algebraic geometry. On the analytic side, one can associate a multiplier ideal to a plurisubharmonic function, a singular Hermitian metric etc. On the algebraic side, one can define multiplier ideal for a \mathbb{Q} -effective divisor, an ideal sheaf or a linear series. I'll discuss these concepts, their relationship and some applications.

4. Title: On the intrinsic derivatives of holomorphic mappings

Speaker: Prof. Zhang Liyou (张利友) (Capital Normal University).

Time: March 13 (Thursday), 14:00—16:00.

Place: Room 709 at Si Yuan Building.

Abstract: We will talk about the definition of intrinsic derivatives of holomorphic mappings introduced by Prof. Q.-K. Lu in 1979. Some applications will also be mentioned.

Special Semester in Several Complex Variables

Schedule (Third Week)

Regular Seminar:

Title: Geometric results and questions related to fake projective planes, fake projective spaces and fake compact Hermitian symmetric spaces.

Speaker: Prof. Sai Kee Yeung (Dept. of Math., Univ. of Purdue).

Time: March 18 (Tuesday), 9:30—11:30.

Place: Room 712 of Si Yuan Building.

Abstract: The main purpose of the talk is to explain a joint work with Gopal Prasad on classification of fake projective planes, fake projective spaces and fake projective compact Hermitian symmetric spaces. Fake projective plane was first introduced by David Mumford. It is defined to be a complex surface with the same Betti numbers as the complex projective plane, and has the smallest possible Euler number among all smooth surfaces of general type. The corresponding manifolds in higher dimensions are called fake projective spaces and fake compact Hermitian symmetric spaces. We would mention the methods involved in the classification of

such manifolds. We would also explain some of the problems encountered and remained, and some of the applications of such investigations.

Special Semester in Several Complex Variables

Schedule (Fifth Week)

Regular Seminar:

1. Title: On the curvature estimates of bounded domains.

Speaker: Prof. Liyou Zhang (张利友) (Capital Normal University).

Time: March 31(Monday), 15:00—17:00.

Place: Room 703 of Si Yuan Building.

Abstract: I will talk about the lower and upper bounds of the holomorphic sectional curvature and Ricci curvature of bounded domains, which was given recently by Prof. Lu using the method of minimal function.

2. Title: Pluripolar sets and convergence sets.

Speaker: Prof. Daowei Ma (马道玮) (Wichita State University).

Time: April 1 (Tuesday), 9:30—11:30.

Place: Room 509 of Si Yuan Building.

Abstract: I will present details of proof of the results of my joint work with T. Neelon. The convergence set of a divergent formal power series $f(x_0, \dots, x_m)$ is the set of all “directions” $\xi \in \mathbb{P}^m$ along which f is absolutely convergent. We prove that every countable union of closed complete pluripolar sets in \mathbb{P}^m is the convergence set of some divergent series .

3. Title: Multiplier ideal sheaf in analytic and algebraic geometry (III).

Speaker: Dr. Jun Li (李军) (Peking University).

Time: April 2 (Wednesday), 15:00—17:00.

Place: Room 509 of Si Yuan Building.

Abstract: Multiplier ideal sheaf is a very useful tool in analytic and algebraic geometry. On the analytic side, one can associate a multiplier ideal to a plurisubharmonic function, a singular Hermitian metric etc. On the algebraic side, one can define multiplier ideal for a \mathbb{Q} -effective divisor, an ideal sheaf or a linear series. I'll discuss these concepts, their relationship and some applications.

Working Seminar:

4. Title: Positivity of vector bundles and vanishing theorems (II).

Speaker: Xiankui Meng (孟宪奎) (Institute of Math., AMSS, CAS).

Time: April 3 (Thursday), 15:00—17:00.

Place: Room 712 of Si Yuan Building.

Abstract: This is a report of reading book. We first recall a treatment in complex differential geometry about the Bochner-Kodaira-Nakano identity, then discuss the positivity concepts for vector bundles. Based on these preparations and the basic results from Hodge theory, proofs of a few vanishing theorems for Hermitian vector bundles will be given.

Special Semester in Several Complex Variables

Schedule (Sixth Week)

Regular Seminar:

1. Title: Non existence of smooth Levi flat hypersurfaces in the complex projective space: known results and open problems.

Speaker: Prof. Andrei Iordan (University of Paris 6).

Time: April 8 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: In 1993, D. Cerveau conjectured the non existence of smooth Levi flat real hypersurfaces (i.e. of real hypersurfaces admitting a local foliation by complex analytic hypersurfaces) in the complex n -dimensional projective space $\mathbb{C}P^n$, $n \geq 2$. If $n \geq 3$, A. Lins Neto proved in 1999 the non existence of real analytic Levi flat hypersurfaces in $\mathbb{C}P^n$ and Y.-T. Siu proved in 2000 the non existence of $C^{1,2}$ smooth Levi flat hypersurfaces in $\mathbb{C}P^n$. The conjecture is still open in dimension 2.

The techniques of the proofs of the non existence of real analytic smooth Levi flat hypersurfaces in $\mathbb{C}P^n$, $n \geq 3$, are different from the smooth case.

In this lecture, we will discuss these techniques and the obstructions which arise in dimension 2.

2. Title: Multiplier ideal sheaf in analytic and algebraic geometry (IV).

Speaker: Dr. Jun Li (李军) (Peking University).

Time: April 9 (Wednesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: Multiplier ideal sheaf is a very useful tool in analytic and algebraic geometry. On the analytic side, one can associate a multiplier ideal to a plurisubharmonic function, a singular Hermitian metric etc. On the algebraic side, one can define multiplier ideal for a \mathbb{Q} -effective divisor, an ideal sheaf or a linear series. I'll discuss these concepts, their relationship and some applications.

3. Title: Deformations of Levi flat structures.

Speaker: Prof. Andrei Iordan (University of Paris 6).

Time: April 10 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: A smooth real hypersurface in a complex manifold is Levi flat if admits a foliation by complex hypersurfaces. The existence and properties of Levi flat hypersurfaces in compact complex manifolds are very important in the geometric theory of foliations and were intensively studied last years.

In this lecture, we will give firstly a deformation theory for integrable distributions of codimension 1 by defining a differential graduated Lie algebra and a Maurer-Cartan equation associated to the foliation. We find than a natural parametrization adapted for deformations of smooth Levi flat hypersurfaces in a compact complex manifold and study cases of rigidity, i.e. when the moduli space of deformations has no non-trivial curves through the origin. Then, we will give an intrinsic theory of deformations of abstract Levi flat structures.

This is a report of joint works in collaboration with Paolo de Bartolomeis.

Special Semester in Several Complex Variables

Schedule (Seventh Week)

Regular Seminar:

1. Title: Bergman Kernel asymptotics for lower energy forms.

Speaker: Prof. Chin-Yu Hsiao (Institute of Mathematics, Academia Sinica, Taipei).

Time: April 14 (Monday), 9:30—11:30.

Place: Room 212 of South Building.

Abstract: In my work with Marinescu (Asymptotics of spectral function of lower energy forms and Bergman kernel of semi-positive and big line bundles, 101 pages, to appear in CAG), we give for the first time a microlocal study of the complex Witten Laplacian. As an application, we obtain a full asymptotic expansion of the spectral function corresponding to the lower part of the spectrum of the Kodaira Laplacian on high tensor powers of a holomorphic line bundle. From this result, we could deduce many classical results in complex geometry (e.g. Kodaira embedding and vanishing Theorems, Demailly's Morse inequalities, Bergman kernel asymptotics for ample line bundles...). In this talk, I will explain how to obtain these classical results from this result. If time is enough, I will also mention a new result obtained in this work: the existence of the full asymptotics expansion for Bergman kernel for a big line bundle twisted with a multiplier ideal sheaf. As a corollary, we could reprove the Shiffman conjecture, asserting that Moishezon manifolds can be characterized in terms of integral Kähler current.

2. Title: Kernel asymptotics and a pure analytic proof of Kodaira embedding theorem.

Speaker: Prof. Chin-Yu Hsiao (Institute of Mathematics, Academia Sinica, Taipei).

Time: April 16 (Wednesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: In this talk, I will present a proof of the existence of Bergman kernel asymptotic expansions for high power of an ample line bundle and by using these asymptotics I will give a pure analytic proof of Kodaira embedding theorem.

3. Title: Bergman kernels and eigenvalue estimate of $\bar{\partial}$ -Laplacian.

Speaker: Zhiwei Wang (Institute of Mathematics, AMSS, CAS).

Time: April 17 (Thursday), 9:00—11:00.

Place: Room 712 of SiYuan Building.

Abstract: Let (X, ω) be a compact Kähler manifold. Let (L, h) be a hermitian holomorphic line bundle over X , such that $\Theta_{L, h} \geq -\varepsilon \omega$ for a small $\varepsilon > 0$, E be a holomorphic line bundle over X . For $k \in \mathbb{N}_+$, denote by $X_k := (X, k\omega)$ the Kähler manifold X with new scaled metric. Estimates of the number of eigenvalues smaller than λ of the $\bar{\partial}$ -Laplacian on forms on X_k with values in $L^k \otimes E$ are presented for $0 \leq \lambda < k$. In particular, when $\lambda = 0$, we get a numeric bound for the cohomology groups.

Special Semester in Several Complex Variables

Schedule (Eighth Week)

Regular Seminar:

1. Title: Scaling Methods in Complex Analytic Geometry (I)
——Basic example of the scaling methods in complex analysis.

Speaker: Prof. Kang-Tae Kim (Center for Geometry and its Applications, POSTECH, South Korea).

Time: April 21 (Monday), 15:00—16:00.

Place: Room 210 of South Building.

Abstract: I will start slowly; I shall begin with finding the well-known conformal equivalence between the open unit disc and the upper half plane, but by the scaling method. This will immediately show how to generalize the trivial example to a not-so-trivial statement, thus demonstrating the power of the scaling method in complex analysis. Then we shall contemplate upon the list of possible problems that can be solved exploiting such ideas.

2. Title: Scaling Methods in Complex Analytic Geometry (II)
——Domains with non-compact automorphism group.

Speaker: Prof. Kang-Tae Kim (Center for Geometry and its Applications, POSTECH, South Korea).

Time: April 21 (Monday), 16:00—17:00.

Place: Room 210 of South Building.

Abstract: Over the years, three representing theorems pertaining to the classification of bounded/hyperbolic pseudoconvex domains with noncompact automorphism group: Wong-Rosay theorem, Bedford-Pinchuk theorem, Bidisc theorem (by this lecturer). I shall explain how these theorems are proved, and what further generalizations and related studies have been performed. In summary the main theme of research in this line is in the following question: "Which domains admit non-compact automorphism group?"

3. Title: Integral Kernel Methods for Weakly Pseudoconvex Domains (I).

Speaker: Prof. R. Michael Range (State University of New York at Albany).

Time: April 22 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: In complex analysis in ONE variable, the classical Cauchy kernel has numerous important applications, so it is an important topic in multidimensional complex analysis to study corresponding higher dimensional kernels and applications. We shall begin this course with a quick review of such well known generalizations and results. We will then cover in detail a recent kernel construction that is valid on arbitrary smoothly bounded *weakly* (that is, **not necessarily strictly**) pseudoconvex domains. In that generality it is not possible to construct explicit kernels that are holomorphic in the parameter. Instead, the goal is to preserve some estimates that reflect the complex geometry of the boundary and the special role of the differentiations with respect to the *complex conjugate* variables. We will discuss some basic properties of the new kernel and use it in a general integral representation formula for $(0, q)$ forms to obtain some pointwise a-priori estimates that are the analogue of the basic estimate on pseudoconvex domains in the L^2 theory of the complex Neumann problem.

4. Title: Scaling Methods in Complex Analytic Geometry (III)

——Stretching coordinates and Bergman geometry.

Speaker: Prof. Kang-Tae Kim (Center for Geometry and its Applications, POSTECH, South Korea).

Time: April 23 (Wednesday), 9:30—10:30.

Place: Room 210 of South Building.

Abstract: Scaling method, initiated by S. I. Pinchuk (in the late 1970's) consists of three components: (1) non-compact automorphism group, (2) stretching sequence, and (3) the centering process. Although the method was developed initially to understand the domains with non-compact automorphism group, it has turned out that (2) and (3) without (1) is still quite useful—which can obtain results on the boundary behavior of holomorphic (pseudo)invariants. This method can be used as a supplementary method to the traditional micro-local analysis. I will demonstrate some of these in the context of Bergman geometry.

5. Title: Scaling Methods in Complex Analytic Geometry (IV)

—Various applications of Scaling methods and open problems.

Speaker: Prof. Kang-Tae Kim (Center for Geometry and its Applications, POSTECH, South Korea).

Time: April 23 (Wednesday), 10:30—11:30.

Place: Room 210 of South Building.

Abstract: The scaling method is quite flexible. As an application, I shall sketch the analysis of the squeezing function (in the sense of Liu-Sun-Yau and Yeung) which I obtained about a year ago in a collaboration with Liyou Zhang of CNU. Another example is my recent work with Robert E. Greene of UCLA on the semicontinuity phenomenon of automorphism groups (simplifying and generalizing the classic results by Greene-Krantz in the early 1980's). I shall conclude the series suggesting some open problems.

6. Title: Integral Kernel Methods for Weakly Pseudoconvex Domains (II).

Speaker: Prof. R. Michael Range (State University of New York at Albany).

Time: April 24 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: In complex analysis in ONE variable, the classical Cauchy kernel has numerous important applications, so it is an important topic in multidimensional complex analysis to study corresponding higher dimensional kernels and applications. We shall begin this course with a quick review of such well known generalizations and results. We will then cover in detail a recent kernel construction that is valid on arbitrary smoothly bounded *weakly* (that is, **not necessarily strictly**) pseudoconvex domains. In that generality it is not possible to construct explicit kernels that are holomorphic in the parameter. Instead, the goal is to preserve some estimates that reflect the complex geometry of the boundary and the special role of the differentiations with respect to the *complex conjugate* variables. We will discuss some basic properties of the new kernel and use it in a general integral representation formula for $(0, q)$ forms to obtain some pointwise a-priori estimates that are the analogue of the basic estimate on pseudoconvex domains in the L^2 theory of the complex Neumann problem.

Special Semester in Several Complex Variables

Schedule (Ninth Week)

Regular Seminar:

1. Title: Positive Closed Currents and Dynamics (I).

Speaker: Prof. Nessim Sibony (University of Paris XI).

Time: April 28 (Monday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: One of the main tools in holomorphic dynamics in several complex variables is pluripotential theory. More precisely it is the theory of positive closed or dd^c closed currents. The course will give an introduction to that theory .

1. Notions of positivity, slicing, pull-back, extension results.
2. Examples of currents from dynamics. Useful spaces of currents.
3. Application to Green currents and entropy.

2. Title: Integral Kernel Methods for Weakly Pseudoconvex Domains (III).

Speaker: Prof. R. Michael Range (State University of New York at Albany).

Time: April 29 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: In complex analysis in ONE variable, the classical Cauchy kernel has numerous important applications, so it is an important topic in multidimensional complex analysis to study corresponding higher dimensional kernels and applications. We shall begin this course with a quick review of such well known generalizations and results. We will then cover in detail a recent kernel construction that is valid on arbitrary smoothly bounded *weakly* (that is, **not necessarily strictly**) pseudoconvex domains. In that generality it is not possible to construct explicit kernels that are holomorphic in the parameter. Instead, the goal is to preserve some estimates that reflect the complex geometry of the boundary and the special role of the differentiations with respect to the *complex conjugate* variables. We will discuss some basic properties of the new kernel and use it in a general integral representation formula for $(0,q)$ forms to obtain some pointwise a-priori estimates that are the analogue of the basic estimate on pseudoconvex domains in the L^2 theory of the complex Neumann problem.

3. Title: Positive Closed Currents and Dynamics (II).

Speaker: Prof. Nessim Sibony (University of Paris XI).

Time: April 30 (Wednesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: One of the main tools in holomorphic dynamics in several complex variables is pluripotential theory. More precisely it is the theory of positive closed or dd^c closed currents. The course will give an introduction to that theory .

1. Notions of positivity, slicing, pull-back, extension results.
2. Examples of currents from dynamics. Useful spaces of currents.
3. Application to Green currents and entropy.

4. Title: Integral Kernel Methods for Weakly Pseudoconvex Domains (IV).

Speaker: Prof. R. Michael Range (State University of New York at Albany).

Time: April 30 (Wednesday), 15:00—17:00.

Place: Room 204 of South Building.

Abstract: In complex analysis in ONE variable, the classical Cauchy kernel has numerous important applications, so it is an important topic in multidimensional complex analysis to study corresponding higher dimensional kernels and applications. We shall begin this course with a quick review of such well known generalizations and results. We will then cover in detail a recent kernel construction that is valid on arbitrary smoothly bounded *weakly* (that is, **not necessarily strictly**) pseudoconvex domains. In that generality it is not possible to construct explicit kernels that are holomorphic in the parameter. Instead, the goal is to preserve some estimates that reflect the complex geometry of the boundary and the special role of the differentiations with respect to the *complex conjugate* variables. We will discuss some basic properties of the new kernel and use it in a general integral representation formula for $(0,q)$ forms to obtain some pointwise a-priori estimates that are the analogue of the basic estimate on pseudoconvex domains in the L^2 theory of the complex Neumann problem.

Special Semester in Several Complex Variables Schedule (Tenth Week)

Regular Seminar:

1. Title: Positive Closed Currents and Dynamics (III).

Speaker: Prof. Nessim Sibony (University of Paris XI).

Time: May 5 (Monday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: One of the main tools in holomorphic dynamics in several complex variables is pluripotential theory. More precisely it is the theory of positive closed or dd^c closed currents. The course will give an introduction to that theory .

1. Notions of positivity, slicing, pull-back, extension results.
2. Examples of currents from dynamics. Useful spaces of currents.
3. Application to Green currents and entropy.

2. Title: A characterization of Carleman sets and an application to products of stratified totally real Carleman sets.

Speaker: Erlend Fornaess Wold (University of Oslo).

Time: May 6 (Tuesday), 9:30—10:30.

Place: Room 210 of South Building.

Abstract: We will consider the following question asked by E. L. Stout: Let $M_j \subset \mathbb{C}^n$ be totally real Carleman-sets for $j=1,2$. Does it follow that $M_1 \times M_2 \subset \mathbb{C}^{2n}$ is a Carleman set? The answer is yes. Recall that a closed subset $M \subset \mathbb{C}^n$ is said to be a Carleman set, if for any continuous function $f \in \mathcal{C}(M)$ and any strictly positive continuous function $\varepsilon \in \mathcal{C}(M)$, there exists an entire function $g \in \mathcal{O}(\mathbb{C}^n)$ such that $|f(z) - g(z)| < \varepsilon(z)$ for all $z \in M$. The approach is to characterize Carleman sets by two conditions that are easy to verify for products, namely that M is Carleman if and only if

- (i) Any compact subset $K \subset M$ is polynomially convex, and
- (ii) for any compact $L \subset \mathbb{C}^n$ there exists $R > 0$ such that for any compact $K \subset M$ we have that

$$\widehat{L \cup K} \setminus (L \cup K) \subset B_R(0).$$

This is joint work with B.S. Magnusson, and is based on joint work with P.E. Manne and N. Øvrelid.

3. Title: Symplectic completion of jets.

Speaker: Erlend Fornaess Wold (University of Oslo).

Time: May 6 (Tuesday), 10:30—11:30.

Place: Room 210 of South Building.

Abstract: Working on modeling problems in accelerator physics, Dragt and Abel asked the following: given a symplectic d -jet Q at the origin in \mathbb{C}^6 , does there exist a symplectic polynomial map P such that $(P-Q)(z)=O(\|z\|^{d+1})$? Moreover they conjectured a lower bound for the degree of the polynomial P . Recall that a symplectic d -jet Q is a polynomial of degree d such that $(Q^*\omega-\omega)(z)=O(\|z\|^{d-1})$, where ω is the standard symplectic form on \mathbb{C}^6 . We will answer their question in the affirmative, and we will confirm their conjectured degree bound in the physically relevant cases $2\leq d\leq 11$. It turns out however that the conjectured degree bound fails for 3-jets in \mathbb{C}^4 ; this is a consequence of a result of Todd that seven involutive lines in \mathbb{P}^3 always lie in a quartic. This is joint work with E. Løw, H. Peters and J.V. Pereira.

4. Title: Undetermined functions in L^2 extension problems and applications.

Speaker: Dr. Qi'an Guan (Peking University).

Time: May 6 (Tuesday), 15:00—17:00.

Place: Room 202 of South Building.

Abstract: In this talk, we will recall our solutions of L^2 extension problems with optimal estimate by using undetermined functions. Some applications such as our solutions of several related conjectures on open Riemann surfaces will also be mentioned.

5. Title: Positive Closed Currents and Dynamics (IV).

Speaker: Prof. Nessim Sibony (University of Paris XI).

Time: May 7 (Wednesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: One of the main tools in holomorphic dynamics in several complex variables is pluripotential theory. More precisely it is the theory of positive closed or dd^c closed currents. The course will give an introduction to that theory .

1. Notions of positivity, slicing, pull-back, extension results.
2. Examples of currents from dynamics. Useful spaces of currents.
3. Application to Green currents and entropy.

6. Title: Geometry and Topology of the space of Kähler metrics (I).

Speaker: Prof. Vincent Guedj (Paul Sabatier University).

Time: May 8 (Thursday), 9:30—11:30.

Place: Room 202 of South Building.

Abstract: Let X be a compact Kähler manifold. The space of Kähler metrics in a fixed cohomology class can be formally regarded as a non positively curved infinite dimensional locally symmetric space. We shall describe the corresponding Mabuchi L^2 metric, geodesics and characterize its metric completion in terms of finite energy currents.

7. Title: Defining functions and cores of unbounded domains.

Speaker: Prof. Nikolay Shcherbina (Wuppertal University).

Time: May 8 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: We discuss the notion of a plurisubharmonic defining function for an unbounded strictly pseudoconvex domain in a complex manifold. Then we indicate the proof of their existence. Next, we discuss the notion of the core of such domains—the set where all defining function fail to be strictly plurisubharmonic. Finally, we indicate the proof of 1-pseudoconcavity of cores and, in some cases, we describe their structure.

8. Title: Positive Closed Currents and Dynamics (V).

Speaker: Prof. Nessim Sibony (University of Paris XI).

Time: May 9 (Friday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: One of the main tools in holomorphic dynamics in several complex variables is pluripotential theory. More precisely it is the theory of positive closed or dd^c closed currents. The course will give an introduction to that theory .

1. Notions of positivity, slicing, pull-back, extension results.
2. Examples of currents from dynamics. Useful spaces of currents.
3. Application to Green currents and entropy.

9. Title: Geometry and Topology of the space of Kähler metrics (II).

Speaker: Prof. Vincent Guedj (Paul Sabatier University).

Time: May 9 (Friday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: Let X be a compact Kähler manifold. The space of Kähler metrics in a fixed cohomology class can be formally regarded as a non positively curved infinite dimensional locally symmetric space. We shall describe the corresponding Mabuchi L^2 metric, geodesics and characterize its metric completion in terms of finite energy currents.

Special Semester in Several Complex Variables

Schedule (Eleventh Week)

Regular Seminar:

1. Title: Equisingular approximations of quasi-plurisubharmonic functions.

Speaker: Dr. Langfeng Zhu (Wuhan University).

Time: May 13 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: We will discuss the equisingular regularization process of quasi-plurisubharmonic functions.

2. Title: Multiplier ideal sheaf in analytic and algebraic geometry (V).

Speaker: Dr. Jun Li (Peking University).

Time: May 15 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: Multiplier ideal sheaf is a very useful tool in analytic and algebraic geometry. On the analytic side, one can associate a multiplier ideal to a plurisubharmonic function, a singular Hermitian metric etc. On the algebraic side, one can define multiplier ideal for a \mathbb{Q} -effective divisor, an ideal sheaf or a linear series. I'll discuss these concepts, their relationship and some applications.

Working Seminar:

3. Title: Positivity of vector bundles and vanishing theorems (III).

Speaker: Xiankui Meng (Institute of Mathematics, AMSS, CAS).

Time: May 16 (Friday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: This is a report of reading book. We first recall a treatment in complex differential geometry about the Bochner-Kodaira-Nakano identity, then discuss the positivity concepts for vector bundles. Based on these preparations and the basic results from Hodge theory, proofs of a few vanishing theorems for Hermitian vector bundles will be given.

Special Semester in Several Complex Variables

Schedule (Twelfth Week)

Regular Seminar:

1. Title: Oka manifolds (I).

Speaker: Prof. Finnur Lárusson (University of Adelaide).

Time: May 19 (Monday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: The class of Oka manifolds has emerged from Gromov's seminal work on the Oka principle through the work of Forstnerič, the speaker, and others over the past decade. Roughly speaking, Oka manifolds are complex manifolds that are the target of "many" holomorphic maps from affine spaces. They are "dual" to Stein manifolds and "opposite" to Kobayashi-hyperbolic manifolds. The prototypical examples are complex homogeneous spaces, but there are many other examples: there are many ways to construct new Oka manifolds from old. The class of Oka manifolds has good formal properties, partly explained by a close connection with abstract homotopy theory. The following topics will be covered in the lectures:

1. Introduction and motivation.
2. Stein manifolds.
3. Gromov's linearisation method.
4. Forstnerič's theorem.
5. Properties of Oka manifolds. New Oka manifolds from old.
6. Affine simplices in Oka manifolds.
7. The space of holomorphic maps from a Stein manifold to an Oka manifold.
8. Deformations of Oka manifolds.

2. Title: Bifurcations within holomorphic families of polynomials: tools.

Speaker: Prof. François Berteloot (Toulouse Mathematics Institute).

Time: May 20 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: In this talk we will study dynamical bifurcations within holomorphic families of polynomials of one complex variable by mean of ergodic and pluripotential theory. The properties of the Lyapunov exponent of the equilibrium measure will play an essential role. Our main goal is to present a proof of the classical Mane-Sad Sullivan theory which can be adapted for families of polynomial-like maps of several variables.

3. Title: Oka manifolds (II).

Speaker: Prof. Finnur Lárusson (University of Adelaide).

Time: May 22 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: The class of Oka manifolds has emerged from Gromov's seminal work on the Oka principle through the work of Forstnerič, the speaker, and others over the past decade. Roughly speaking, Oka manifolds are complex manifolds that are the target of "many" holomorphic maps from affine spaces. They are "dual" to Stein manifolds and "opposite" to Kobayashi-hyperbolic manifolds. The prototypical examples are complex homogeneous spaces, but there are many other examples: there are many ways to construct new Oka manifolds from old. The class of Oka manifolds has good formal properties, partly explained by a close connection with abstract homotopy theory. The following topics will be covered in the lectures:

1. Introduction and motivation.
2. Stein manifolds.
3. Gromov's linearisation method.
4. Forstnerič's theorem.
5. Properties of Oka manifolds. New Oka manifolds from old.
6. Affine simplices in Oka manifolds.
7. The space of holomorphic maps from a Stein manifold to an Oka manifold.
8. Deformations of Oka manifolds.

4. Title: Bifurcations within holomorphic families of polynomials: classical results and recent developments.

Speaker: Prof. François Berteloot (Toulouse Mathematics Institute).

Time: May 23 (Friday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: In this talk we will study dynamical bifurcations within holomorphic families of polynomials of one complex variable by mean of ergodic and pluri-potential theory. The properties of the Lyapunov exponent of the equilibrium measure will play an essential role. Our main goal is to present a proof of the classical Mane-Sad Sullivan theory which can be adapted for families of polynomial-like maps of several variables.

Special Semester in Several Complex Variables

Schedule (Thirteenth Week)

Regular Seminar:

1. Title: Oka manifolds (III).

Speaker: Prof. Finnur Lárusson (University of Adelaide).

Time: May 26 (Monday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: The class of Oka manifolds has emerged from Gromov's seminal work on the Oka principle through the work of Forstnerič, the speaker, and others over the past decade. Roughly speaking, Oka manifolds are complex manifolds that are the target of "many" holomorphic maps from affine spaces. They are "dual" to Stein manifolds and "opposite" to Kobayashi-hyperbolic manifolds. The prototypical examples are complex homogeneous spaces, but there are many other examples: there are many ways to construct new Oka manifolds from old. The class of Oka manifolds has good formal properties, partly explained by a close connection with abstract homotopy theory. The following topics will be covered in the lectures:

1. Introduction and motivation.
2. Stein manifolds.
3. Gromov's linearisation method.
4. Forstnerič's theorem.
5. Properties of Oka manifolds. New Oka manifolds from old.
6. Affine simplices in Oka manifolds.
7. The space of holomorphic maps from a Stein manifold to an Oka manifold.
8. Deformations of Oka manifolds.

2. Title: Extension of plurisubharmonic functions with growth control.

Speaker: Prof. Dan Coman (Syracuse University).

Time: May 27 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: This is joint work with Vincent Guedj and Ahmed Zeriahi.

Let $X \subset \mathbb{C}^n$ be a (closed) analytic subvariety. Coltoiu proved that a plurisubharmonic (psh) function on X extends to a psh function on \mathbb{C}^n . We show here that it is possible to obtain extensions with global growth control:

Theorem A. *Let X be an analytic subvariety of a Stein manifold M and let φ be a psh function on X . Assume that u is a continuous psh exhaustion function on M so that $\varphi(z) < u(z)$ for all $z \in X$. Then for every $c > 1$ there exists a psh function $\psi = \psi_c$ on M so that $\psi|_X = \varphi$ and $\psi(z) < c \max\{u(z), 0\}$ for all $z \in M$.*

We then look at a similar problem on a compact Kähler manifold V . Given a Kähler form ω , let

$$PSH(V, \omega) = \{\varphi \in L^1(V, [-\infty, +\infty)) : \varphi \text{ upper semicontinuous, } dd^c \varphi \geq -\omega\}$$

denote the set of ω -plurisubharmonic (ω -psh) functions. If $X \subset V$ is an analytic subvariety, the class $PSH(X, \omega|_X)$ of $\omega|_X$ -psh functions on X is defined as follows: a function $\varphi: X \rightarrow [-\infty, +\infty)$ is called $\omega|_X$ -psh if $\varphi \not\equiv -\infty$ on X and if there exist an open cover $\{U_i\}_{i \in I}$ of X and psh functions φ_i, ρ_i defined on U_i , where ρ_i is smooth and $dd^c \rho_i = \omega$, so that $\rho_i + \varphi = \varphi_i$ holds on $X \cap U_i$, for every $i \in I$.

By restriction, ω -psh functions on V yield $\omega|_X$ -psh functions on X . If ω is a *Hodge form*, i.e. a Kähler form with integer cohomology class, our second result is:

Theorem B. *Let X be a subvariety of a projective manifold V equipped with a Hodge form ω . Then any $\omega|_X$ -psh function on X is the restriction of an ω -psh function on V .*

We conclude by discussing the extension problem for psh functions in the Lelong classes. If X is an analytic subvariety of \mathbb{C}^n and $\gamma > 0$, we denote by $\mathcal{L}_\gamma(X)$ (and by $\mathcal{L}(X)$ when $\gamma=1$) the *Lelong class* of psh functions φ on X which verify $\varphi(z) \leq \gamma \log^+ \|z\| + O(1)$ on X . By Theorem A, functions in $\mathcal{L}(X)$ admit a psh extension in each class $\mathcal{L}_\gamma(\mathbb{C}^n)$, for every $\gamma > 1$. If X is an *algebraic subvariety* of \mathbb{C}^n , Theorem B allows to give a simple characterization of the functions in $\mathcal{L}(X)$ which have an extension in the class $\mathcal{L}(\mathbb{C}^n)$.

3. Title: Oka manifolds (IV).

Speaker: Prof. Finnur Lárusson (University of Adelaide).

Time: May 28 (Wednesday), 15:00—17:00.

Place: Room 202 of South Building.

Abstract: The class of Oka manifolds has emerged from Gromov's seminal work on the Oka principle through the work of Forstnerič, the speaker, and others over the past decade. Roughly speaking, Oka manifolds are complex manifolds that are the target of "many" holomorphic maps from affine spaces. They are "dual" to Stein manifolds and "opposite" to Kobayashi-hyperbolic manifolds. The prototypical examples are complex homogeneous spaces, but there are many other examples: there

are many ways to construct new Oka manifolds from old. The class of Oka manifolds has good formal properties, partly explained by a close connection with abstract homotopy theory. The following topics will be covered in the lectures:

1. Introduction and motivation.
2. Stein manifolds.
3. Gromov's linearisation method.
4. Forstnerič's theorem.
5. Properties of Oka manifolds. New Oka manifolds from old.
6. Affine simplices in Oka manifolds.
7. The space of holomorphic maps from a Stein manifold to an Oka manifold.
8. Deformations of Oka manifolds.

4. Title: Growth of polynomials along transcendental curves in \mathbb{C}^2 .

Speaker: Prof. Dan Coman (Syracuse University).

Time: May 29 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: We begin by recalling a few concepts from pluripotential theory such as the pluricomplex Green function of a non-pluripolar compact set K in \mathbb{C}^k , and the related Bernstein-Walsh inequality which estimates $|P(z)|$ in terms of the uniform norm $\|P\|_K$ and the degree of P , where P is a complex polynomial on \mathbb{C}^k .

We then look at the case when K is a special pluripolar set, namely a disc contained in the graph in \mathbb{C}^2 of an entire transcendental function f on \mathbb{C} . A certain Bernstein-Walsh inequality still holds for such K , and it can be applied to obtain estimates on the growth rate of the functions $P(z, f(z))$ in terms of the degree of P , where $P(z, w)$ is a complex polynomial on \mathbb{C}^2 . This leads to a bound on the maximal number of zeros of $P(z, f(z))$ in a disc $\{|z| \leq r\}$, as P is an arbitrary polynomial of fixed degree. The latter can be viewed as a transcendental Bézout type inequality, bounding in terms of n the maximal number of intersection points over a fixed disc between the transcendental curve $\Gamma = \{(z, f(z)): z \in \mathbb{C}\}$ given by the graph of f and an algebraic curve of degree n in \mathbb{C}^2 . This has applications to number theory.

We survey joint results with E. Poletsky obtained in the case when f is an entire function of finite order, and in the case when graphs are replaced by exponential curves $\Gamma = \{(e^\zeta, e^{\alpha\zeta}): \zeta \in \mathbb{C}\}$, where $\alpha \in (0, 1) \setminus \mathbb{Q}$.

Special Semester in Several Complex Variables

Schedule (Fourteenth Week)

Working Seminar:

1. Title: Positivity of vector bundles and vanishing theorems (IV).

Speaker: Xiankui Meng (Institute of Mathematics, AMSS, CAS).

Time: June 3 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: This is a report of reading book. We first recall a treatment in complex differential geometry about the Bochner-Kodaira-Nakano identity, then discuss the positivity concepts for vector bundles. Based on these preparations and the basic results from Hodge theory, proofs of a few vanishing theorems for Hermitian vector bundles will be given.

Regular Seminar:

2. Title: Multiplier ideal sheaf in analytic and algebraic geometry (VI).

Speaker: Dr. Jun Li (Peking University).

Time: June 5 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: Multiplier ideal sheaf is a very useful tool in analytic and algebraic geometry. On the analytic side, one can associate a multiplier ideal to a plurisubharmonic function, a singular Hermitian metric etc. On the algebraic side, one can define multiplier ideal for a \mathbb{Q} -effective divisor, an ideal sheaf or a linear series. I'll discuss these concepts, their relationship and some applications.

3. Title: Regularity of local CR embedding problem (I).

Speaker: Prof. Xianghong Gong (University of Wisconsin, Madison).

Time: June 6 (Friday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: We will start with the Henkin homotopy formula and $\bar{\partial}_b$ 1/2-gain estimates. We will discuss the integrability of CR vector bundle and the local CR embedding problem. We will demonstrate applications of Nash-Moser rapid iteration technique.

Special Semester in Several Complex Variables

Schedule (Fifteenth Week)

Regular Seminar:

1. Title: Regularity of local CR embedding problem (II).

Speaker: Prof. Xianghong Gong (University of Wisconsin, Madison).

Time: June 10 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: We will start with the Henkin homotopy formula and $\bar{\partial}_b$ 1/2-gain estimates. We will discuss the integrability of CR vector bundle and the local CR embedding problem. We will demonstrate applications of Nash-Moser rapid iteration technique.

2. Title: Regularity of local CR embedding problem (III).

Speaker: Prof. Xianghong Gong (University of Wisconsin, Madison).

Time: June 12 (Thursday), 19:30—21:30.

Place: Room 210 of South Building.

Abstract: We will start with the Henkin homotopy formula and $\bar{\partial}_b$ 1/2-gain estimates. We will discuss the integrability of CR vector bundle and the local CR embedding problem. We will demonstrate applications of Nash-Moser rapid iteration technique.

Special Semester in Several Complex Variables

Schedule (Sixteenth Week)

Regular Seminar:

- 1. Title:** Hardy spaces and BMO.
Speaker: Prof. Kehe Zhu (State University of New York, Albany).
Time: June 16 (Monday), 15:00—17:30.
Place: Room 210 of South Building.
- 2. Title:** Bergman spaces.
Speaker: Prof. Kehe Zhu (State University of New York, Albany).
Time: June 17 (Tuesday), 9:30—11:30.
Place: Room 210 of South Building.
- 3. Title:** Regularity of local CR embedding problem (IV).
Speaker: Prof. Xianghong Gong (University of Wisconsin, Madison).
Time: June 18 (Wednesday), 9:30—11:30.
Place: Room 210 of South Building.

Abstract: We will start with the Henkin homotopy formula and $\bar{\partial}_b$ 1/2-gain estimates. We will discuss the integrability of CR vector bundle and the local CR embedding problem. We will demonstrate applications of Nash-Moser rapid iteration technique.
- 4. Title:** Lipschitz spaces.
Speaker: Prof. Kehe Zhu (State University of New York, Albany).
Time: June 18 (Wednesday), 15:00—17:00.
Place: Room 902 of South Building.
- 5. Title:** Möbius invariant spaces.
Speaker: Prof. Kehe Zhu (State University of New York, Albany).
Time: June 19 (Thursday), 15:00—17:00.
Place: Room 210 of South Building.
- 6. Title:** Fock spaces.
Speaker: Prof. Kehe Zhu (State University of New York, Albany).
Time: June 20 (Friday), 15:00—17:00.
Place: Room 210 of South Building.

Special Semester in Several Complex Variables

Schedule (Eighteenth Week)

Regular Seminar:

1. Title: The Diederich-Fornaess exponent and its applications (I).

Speaker: Prof. Siqi Fu (Rutgers University-Camden).

Time: July 3 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: Diederich and Fornaess showed in 1977 that for any bounded pseudoconvex domain Ω with C^2 boundary in a Stein manifold, there exist a positive constant η and a defining function r such that $-(-r)^\eta$ is pluri-subharmonic on Ω . The Diederich-Fornaess exponent η has found important applications in several complex variables. In these expository lectures, we study the Diederich-Fornaess exponent and its various applications. In the first lecture, we will focus on estimates of the Diederich-Fornaess exponent and the application to L^2 -estimates of the Cauchy-Riemann operator. In the second lecture, we will discuss the applications to regularity theory in the complex Neumann problem and to non-existence of Stein domains with Levi-flat boundaries in complex manifolds. These lectures are based partly on joint work with Mei-Chi Shaw.

2. Title: The Diederich-Fornaess exponent and its applications (II).

Speaker: Prof. Siqi Fu (Rutgers University-Camden).

Time: July 4 (Friday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: Diederich and Fornaess showed in 1977 that for any bounded pseudoconvex domain Ω with C^2 boundary in a Stein manifold, there exist a positive constant η and a defining function r such that $-(-r)^\eta$ is pluri-subharmonic on Ω . The Diederich-Fornaess exponent η has found important applications in several complex variables. In these expository lectures, we study the Diederich-Fornaess exponent and its various applications. In the first lecture, we will focus on estimates of the Diederich-Fornaess exponent and the application to L^2 -estimates of the Cauchy-Riemann operator. In the second lecture, we will discuss the applications to regularity theory in the complex Neumann problem and to non-existence of Stein domains with Levi-flat boundaries in complex manifolds. These lectures are based partly on joint work with Mei-Chi Shaw.

Special Semester in Several Complex Variables

Schedule (Nineteenth Week)

Regular Seminar:

1. Title: Geometry of convex bodies, entropy and gauss curvature flows (I).

Speaker: Prof. Lei Ni (University of California, San Diego).

Time: July 8 (Tuesday), 9:30—11:30.

Place: Room 210 of South Building.

Abstract: Gauss curvature flow was originated by Firey in 1974 to model the tumbling of the stone at the beach. The asymptotic shape was proved to be spherical by B. Andrews in 1999 for surfaces in three dimensional Euclidean space. In this talk I shall discuss the progresses to the problem in high dimensions. Some background on convex bodies and uniqueness via Łojasiewicz inequality shall also be discussed.

2. Title: Geometry of convex bodies, entropy and gauss curvature flows (II).

Speaker: Prof. Lei Ni (University of California, San Diego).

Time: July 10 (Thursday), 15:00—17:00.

Place: Room 210 of South Building.

Abstract: Gauss curvature flow was originated by Firey in 1974 to model the tumbling of the stone at the beach. The asymptotic shape was proved to be spherical by B. Andrews in 1999 for surfaces in three dimensional Euclidean space. In this talk I shall discuss the progresses to the problem in high dimensions. Some background on convex bodies and uniqueness via Łojasiewicz inequality shall also be discussed.